Factorization tests in DVCS and GPD fits

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K. Kumerički, DM, K. Passek-Kumerički (KMP-K), hep-ph/0703179
GPD fits at NLO and NNLO of H1/ZEUS data

KMP-K, 0805.0152 [hep-ph]
constructive critics on ad hoc GPD model approach [lot of good news]
first applications of dispersion integral approach

KMP-K, 0807.0159 [hep-ph]; KM 0904.0458 [hep-ph]
flexible GPD model for small *x* and fits of H1/ZEUS data
dispersion integral fits of HERMES and JLAB data

outline:

- **Photon leptoproduction (DVCS)**
- **GPD properties & representations**
- **Strategies to analyze DVCS data**

  - ad hoc GPD models to provide estimates
  - flexible GPD models: Are we ready? (H1/ZEUS fits)
  - dispersion relation approach (global fit example)

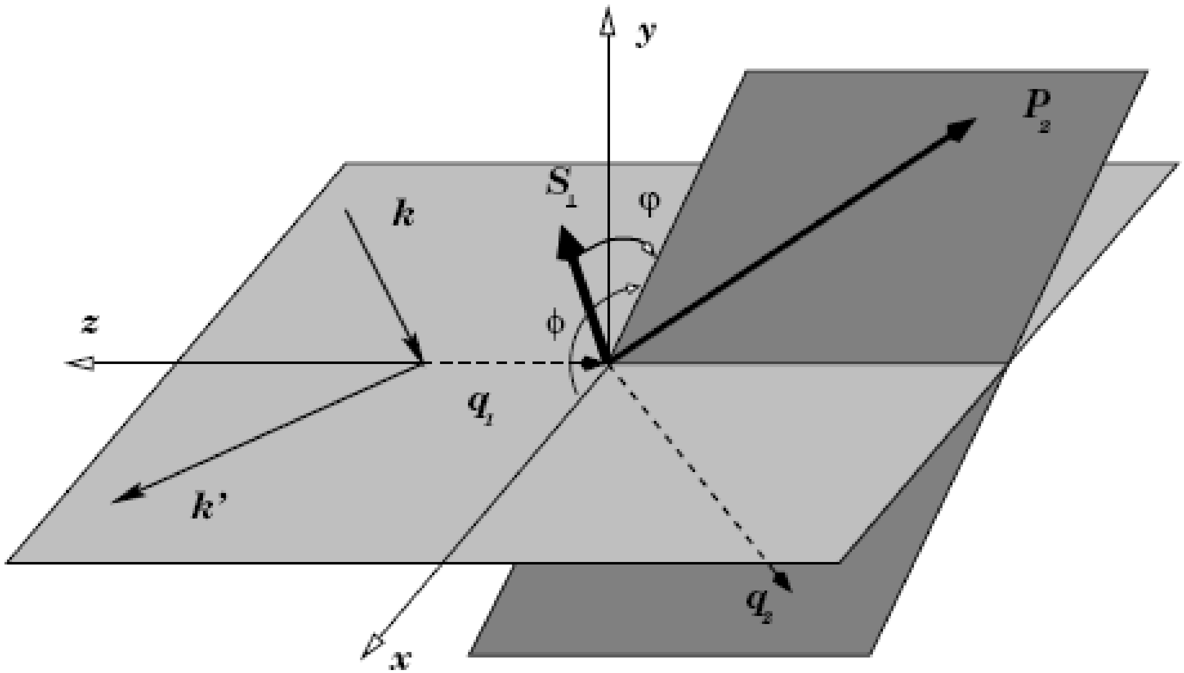
- **Summary**

Photon leptoproduction $e^{\pm}N \to e^{\pm}N\gamma$

measured by ***H1, ZEUS, HERMES, CLAS, HALL A*** collaborations

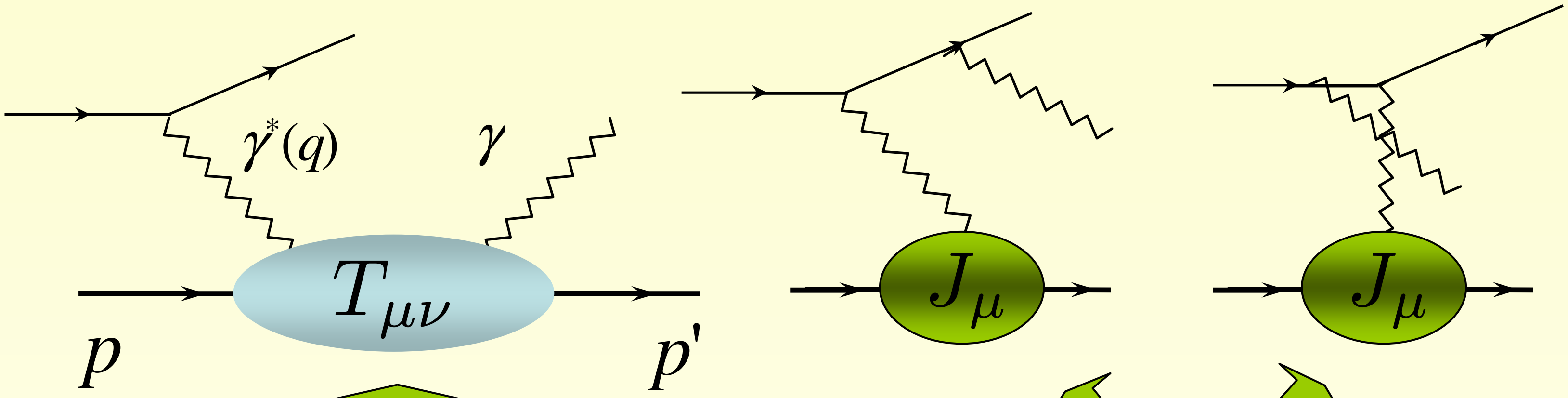
planed at ***COMPASS, JLAB@12GeV***, perhaps at ?? EIC,

$$\frac{d\sigma}{dx_{\mathrm{Bj}}dyd|\Delta^2|d\phi d\varphi} = \frac{\alpha^3 x_{\mathrm{Bj}} y}{16\,\pi^2\,\mathcal{Q}^2}\left(1+\frac{4M^2_{Bj}{}^2}{\mathcal{Q}^2}\right)^{-1/2}\left|\frac{\mathcal{T}}{e^3}\right|^2,$$



$$\begin{aligned} x_{\mathrm{Bj}} &= \frac{\mathcal{Q}^2}{2P_1\cdot q_1} \approx \frac{2\xi}{1+\xi}, \\ y &= \frac{P_1\cdot q_1}{P_1\cdot k}, \\ \Delta^2 &= t \quad (\text{fixed, small}), \\ \mathcal{Q}^2 &= -q_1^2 \quad (> 1\mathrm{GeV}^2), \end{aligned}$$

interference of *DVCS* and *Bethe-Heitler* processes



12 Compton form factors $\mathcal{H}, \mathcal{E}, \widetilde{\mathcal{H}} \cdots$ (helicity amplitudes)

elastic form factors F_1, F_2

$$|\mathcal{T}_{\rm BH}|^2 = \frac{e^6(1+\epsilon^2)^{-2}}{x_{\rm Bj}^2 y^2 \Delta^2 \mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \left\{ c_0^{\rm BH} + \sum_{n=1}^{2} c_n^{\rm BH} \cos(n\phi) \right\},$$

exactly known (LO, QED)

$$|\mathcal{T}_{\rm DVCS}|^2 = \frac{e^6}{y^2 \mathcal{Q}^2} \left\{ c_0^{\rm DVCS} + \sum_{n=1}^{2} \left[c_n^{\rm DVCS} \cos(n\phi) + s_n^{\rm DVCS} \sin(n\phi) \right] \right\},$$

harmonics $\Updownarrow$ <u>**1:1**</u> helicity ampl.

$$\mathcal{I} = \frac{\pm e^6}{x_{\rm Bj} y^3 \Delta^2 \mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \left\{ c_0^{\mathcal{I}} + \sum_{n=1}^{3} \left[c_n^{\mathcal{I}} \cos(n\phi) + s_n^{\mathcal{I}} \sin(n\phi) \right] \right\}.$$

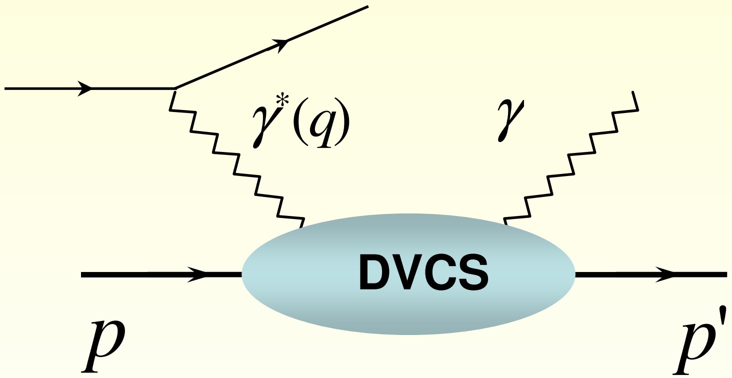
harmonics $\Updownarrow$ **1:1** helicity ampl.

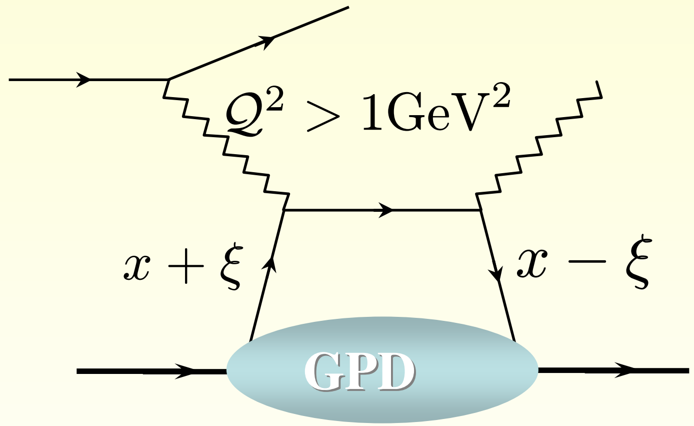
GPDs embed non-perturbative physics

GPDs appear in various hard exclusive processes,

**[DM et. al (90/94)
Radyushkin (96)
Ji (96)]**

e.g., hard electroproduction of photons (DVCS)





$t = \Delta^2$ – fix

$$\mathcal{F}(\xi, \mathcal{Q}^2, t) = \int_{-1}^{1} dx\, C(x, \xi, \alpha_s(\mu), \mathcal{Q}/\mu) F(x, \xi, t, \mu) + O(\tfrac{1}{\mathcal{Q}^2})$$

CFF	**hard scattering part**	**GPD**	**higher twist**
Compton form factor observable	perturbation theory (our conventions/microscope)	universal (conventional)	depends on approximation

relations among **harmonics** and **GPDs** are based on $1/\mathcal{Q}$ expansion:
(all harmonics are expressed by twist-2 and -3 GPDs) **[Diehl et. al (97) Belitsky, DM, Kirchner (01)]**

$$\left\{\begin{matrix} c_1 \\ s_1 \end{matrix}\right\}^{\mathcal{I}} \propto \frac{\Delta}{\mathcal{Q}} \text{tw-2(GPDs)} + O(1/\mathcal{Q}^3), \qquad c_0^{\mathcal{I}} \propto \frac{\Delta^2}{\mathcal{Q}^2} \text{tw-2(GPDs)} + O(1/\mathcal{Q}^4),$$

$$\left\{\begin{matrix} c_2 \\ s_2 \end{matrix}\right\}^{\mathcal{I}} \propto \frac{\Delta^2}{\mathcal{Q}^2}\ \text{tw-3(GPDs)} + O(1/\mathcal{Q}^4), \qquad \left\{\begin{matrix} c_3 \\ s_3 \end{matrix}\right\}^{\mathcal{I}} \propto \frac{\Delta \alpha_s}{\mathcal{Q}} (\text{tw-2})^{\mathrm{T}} + O(1/\mathcal{Q}^3),$$

$$c_0^{\mathrm{CS}} \propto (\text{tw-2})^2, \qquad \left\{\begin{matrix} c_1 \\ s_1 \end{matrix}\right\}^{\mathrm{CS}} \propto \frac{\Delta}{Q} (\text{tw-2})(\text{tw-3}), \qquad \left\{\begin{matrix} c_2 \\ s_2 \end{matrix}\right\}^{\mathrm{CS}} \propto \alpha_s (\text{tw-2})(\text{tw-2})^{\mathrm{GT}}$$

setting up the **perturbative framework:** **[Belitsky, DM (97); Mankiewicz et. al (97); Ji,Osborne (98)]**

- ✓ ***twist-two*** coefficient functions at ***next-to-leading*** order
- ✓ evolution kernels at ***next-to-leading*** order **[Belitsky, DM, Freund (01)]**
- ✓ ***next-to-next-to-leading*** order in a specific conformal subtraction scheme **[KMP-K & Schaefer 06]**
- ✓ ***twist-three*** including quark-gluon-quark correlation at LO **[Anikin,Teryaev, Pire (00); Belitsky DM (00); Kivel et. al]**
- ✓ partial ***twist-three*** sector at ***next-to-leading*** order **[Kivel, Mankiewicz (03)]**
- ✓ `target mass corrections' (not well understood) **[Belitsky DM (01)]**

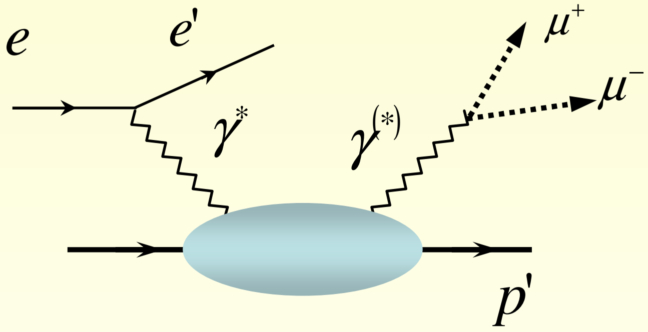
GPD related hard exclusive processes

- Deeply virtual Compton scattering (clean probe)

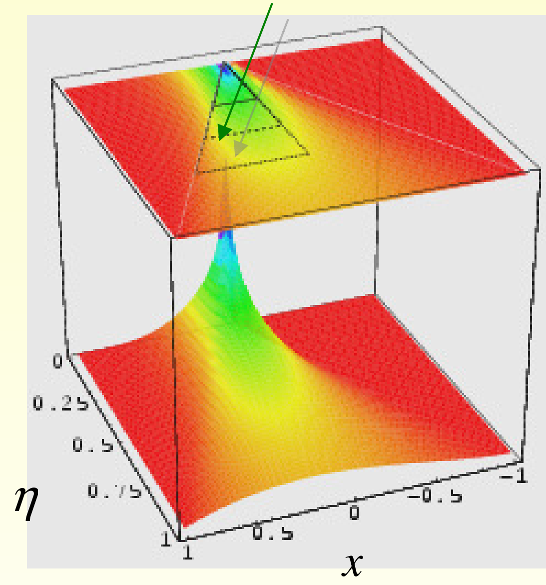
$$ep \to e'p'\gamma$$

$$ep \to e'p'\mu^+\mu^-$$

$$\gamma p \to p'e^+e^-$$



scanned area of the surface as a functions of lepton energy



$ep \to e'\,p'\,\mu^+\mu^-$

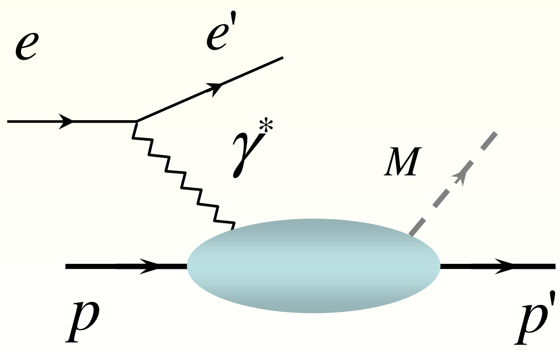
- Hard exclusive meson production (flavor filter)

$$ep \to e'p'\pi$$

$$ep \to e'p'\rho$$

$$ep \to e'n\pi^+$$

$$ep \to e'n\rho^+$$



<u>twist-two observables:</u>

cross sections

transverse target spin asymmetries

- etc.

Can one `measure' GPDs?

- ***CFF*** given as ***GPD*** **convolution**:

$$\mathcal{H}(\xi,t,\mathcal{Q}^2) \stackrel{\mathrm{LO}}{=} \int_{-1}^{1} dx \left(\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon}\right) H(x,\eta=\xi,t,\mathcal{Q}^2)$$

$$\stackrel{\mathrm{LO}}{=} i\pi H^{-}(x=\xi,\eta=\xi,t,\mathcal{Q}^2) + \mathrm{PV}\int_0^1 dx \frac{2x}{\xi^2 - x^2} H^{-}(x,\eta=\xi,t,\mathcal{Q}^2)$$

- *H(x,x,t,$\mathcal{Q}^2$)* viewed as **spectral function** (*s*-channel cut):

$$H^{-}(x,x,t,Q^2) \equiv H(x,x,t,Q^2) - H(-x,x,t,Q^2) \stackrel{\mathrm{LO}}{=} \frac{1}{\pi}\Im\mathrm{m}\mathcal{F}(\xi = x,t,Q^2)$$

[Frankfurt et al (97)
Chen (97)
Terayev (05)
KMP-K (07)
Diehl, Ivanov (07)]

- ***CFFs*** satisfy **dispersion relations**
(not the physical ones, threshold ξ_0 set to 0)

$$\Re\mathrm{e}\mathcal{F}(\xi,t,Q^2) = \frac{1}{\pi}\mathrm{PV}\int_0^1 d\xi' \left(\frac{1}{\xi-\xi'} \mp \frac{1}{\xi+\xi'}\right)\Im\mathrm{m}\mathcal{F}(\xi',t,Q^2) + \mathcal{C}(t,Q^2)$$

[Terayev (05)]

access to the ***GPD*** on the **cross-over line** $\eta = x$ (at LO)

GPD Properties

GPDs are intricate functions: $H(x, \eta = \xi, t, \mu^2 = \mathcal{Q}^2)$

a non-trivial interplay of variable dependence

- *t*-dependence dies out at large *x* (spectator models, indicated by lattice & *X*QS-model)
- effective Regge behavior (from phenomenology) at small *x*; unknown η-dependence
- evolution depends on the GPD shape

at least four phenomenological important GPDs for each parton

GPD-constraints:

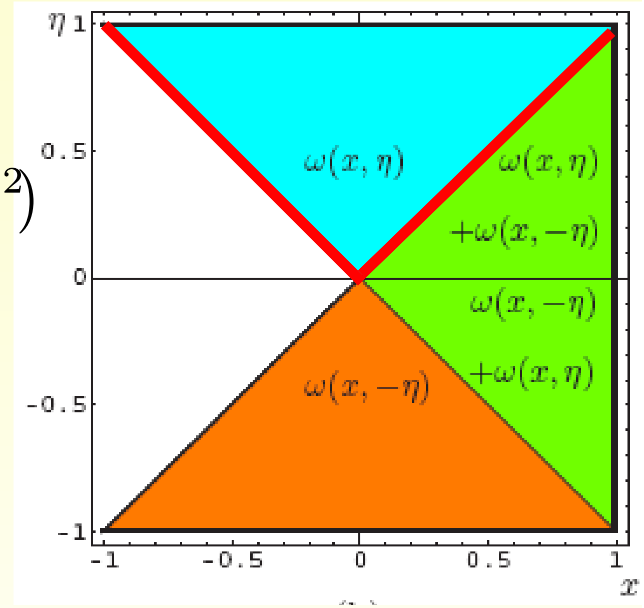
- reduction to PDFs: $$q(x, \mu^2) = \lim_{\Delta \to 0} H(x, \eta, t, \mu^2)$$
- generalized form factor sum rules, e.g.: (polynomiality, GPD support property) $$F_1(t) = \int_{-1}^{1} dx\, H(x, \eta, t, \mu^2)$$
- Ji's sum rule $$\frac{1}{2} = \int_{-1}^{1} dx\, x (H + E)(x, \eta, t = 0, \mu^2)$$
- positivity constraints (**valid at LO**) **[P. Pobylitsa 02]**

  (strongly constraining variable interplay in the outer region)

A partonic duality interpretation

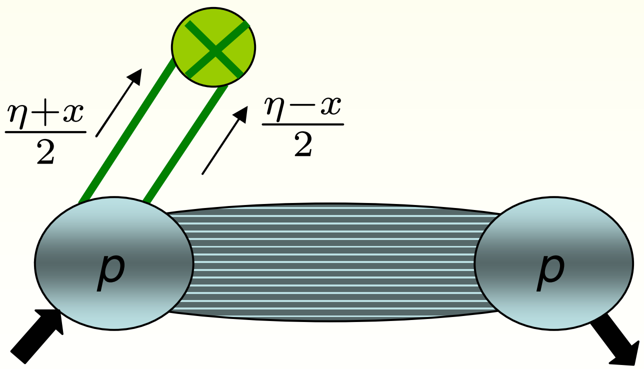
quark GPD (anti-quark $x \to -x$):

$$F = \theta(-\eta \le x \le 1)\, \omega\big(x, \eta, \Delta^2\big) + \theta(\eta \le x \le 1)\, \omega\big(x, -\eta, \Delta^2\big)$$



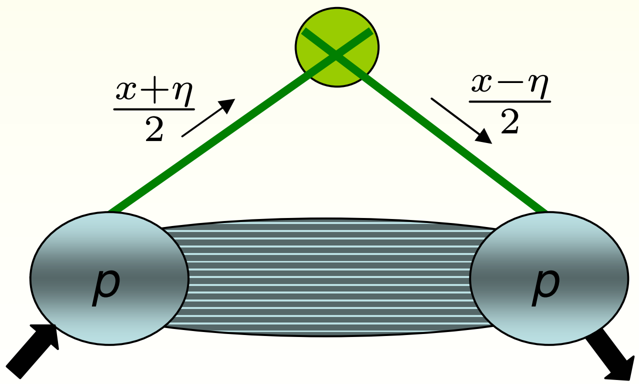
$$\omega\left(x, \eta, \Delta^2\right) = \frac{1}{\eta} \int_0^{\frac{x+\eta}{1+\eta}} dy\, x^p f(y, (x-y)/\eta, \Delta^2)$$

dual interpretation on partonic level:



support extension
is unique [DM et al. 92]

ambiguous (*D*-term)
[DM, A. Schäfer (05)
KMP-K (07)]



central region $-\eta < x < \eta$

mesonic exchange in *t*-channel

outer region $\eta < x$

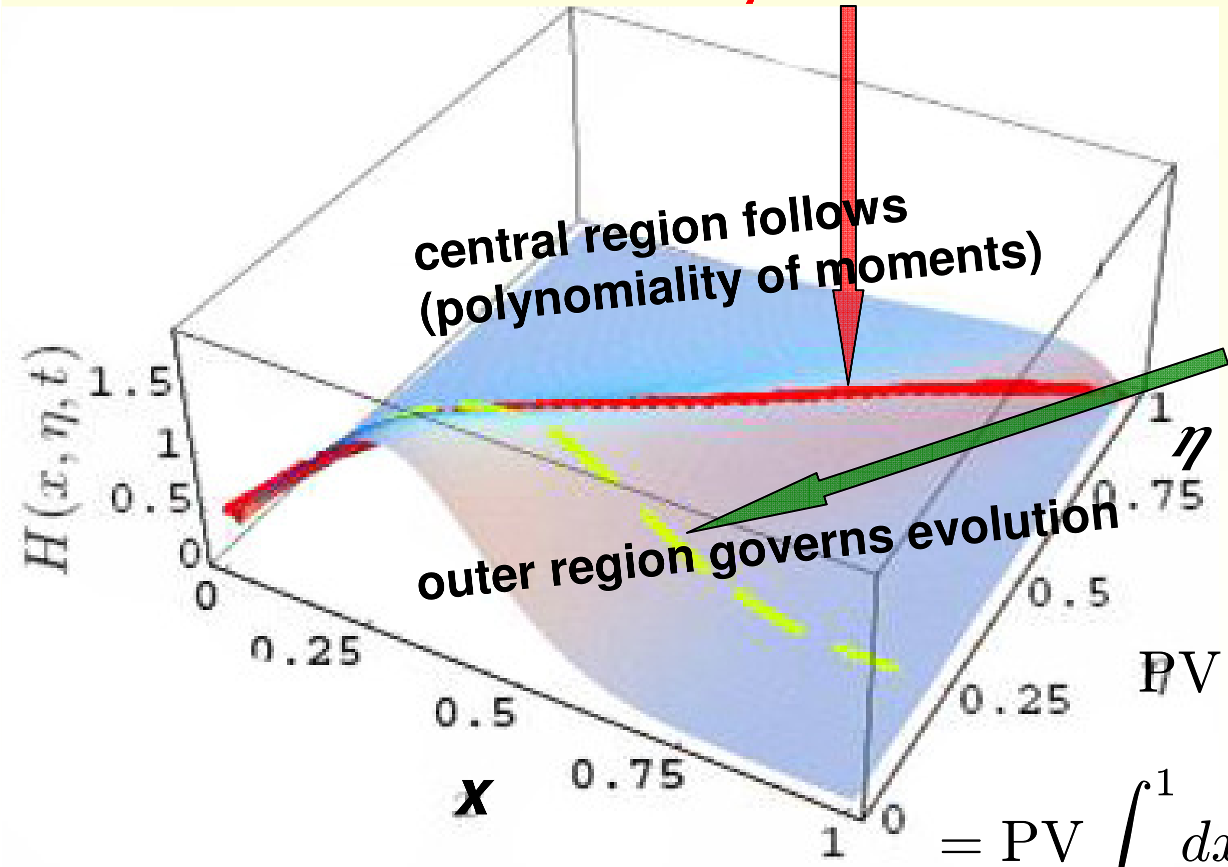
partonic exchange in *s*-channel

Modeling & Evolution

outer region governs the evolution at the cross-over trajectory

$$\mu^2 \frac{d}{d\mu^2} H(x,x,t,\mu^2) = \int_x^1 \frac{dy}{x} V(1, x/y, \alpha_s(\mu)) H(y,x,\mu^2)$$

GPD at $\eta = x$ is `measurable' (LO)



net contribution of outer + central region is governed by a sum rule:

$$\mathrm{PV}\int_0^1 dx\ \frac{2x}{\eta^2 - x^2} H^-(x,\eta,t)$$

$$= \mathrm{PV}\int_0^1 dx\ \frac{2x}{\eta^2 - x^2} H^-(x,x,t) + C(t)$$

Overview: GPD representations

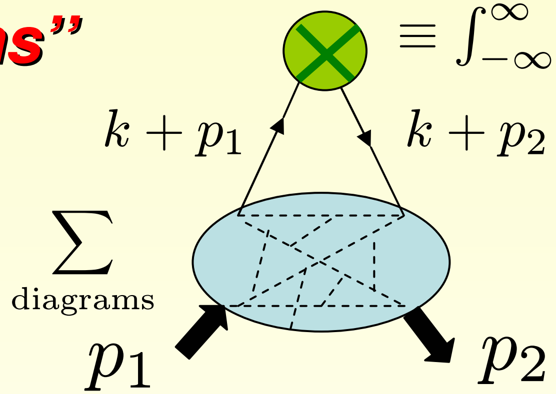
``light-ray spectral functions"

diagrammatic α-representation

**DM, Robaschik, Geyer,
Dittes, Hořejši (88 (92) 94)**

called ***double distributions***

A. Radyushkin (96)



$$\equiv \int_{-\infty}^{\infty} \frac{d\kappa}{2\pi}\, e^{i\kappa(xP^+ - P^+ - 2k^+)}$$

SL(2,R) (conformal) expansion

(series of local operators)

**Radyushkin (97);
Belitsky, Geyer, DM, Schäfer (97);
DM, Schäfer (05); ….**

one version is called Shuvaev transformation,
used in \`dual' (*t*-channel) GPD parameterization

**Shuvaev (99,02); Noritzsch (00)
Polyakov (02,07)**

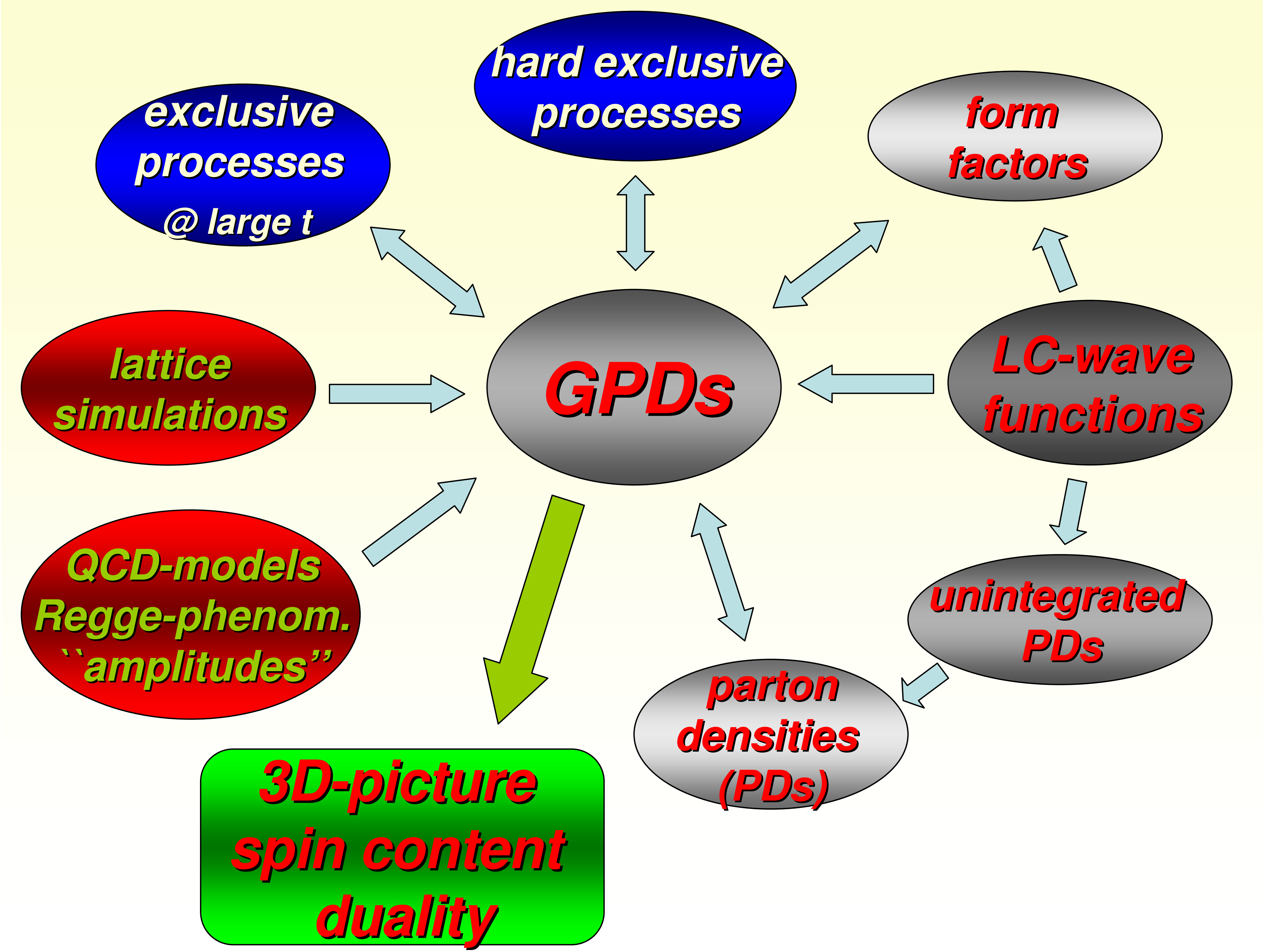
light cone wave function overlap

(Hamiltonian approach in light-cone quantization)

**Diehl, Feldmann,
Jakob, Kroll (98,00)**

**Diehl, Brodsky,
Hwang (00)**

each representation has its own ***advantages***,
however, they are ***equivalent*** **(clearly spelled out in [Hwang, DM 07])**



exclusive processes @ large t
hard exclusive processes
form factors
lattice simulations
GPDs
LC-wave functions
QCD-models
Regge-phenom.
``amplitudes''
unintegrated PDs
parton densities (PDs)
3D-picture
spin content
duality

Strategies to analyze DVCS data

GPD model approach:

ad hoc modeling: **VGG** code [**Goeke et. al (01)** based on **Radyuskin's** DDA]
(first decade) BKM model [**Belitsky, Kirchner, DM (01)** based on RDDA]
`aligned jet' model [**Freund, McDermott, Strikman (02)**]
Kroll/Goloskokov (05) based on RDDA [not utilized for DVCS]
`dual' model [**Polyakov,Shuvaev 02;Guzey,Teckentrup 06;Polyakov 07**]
" -- " [**KMP-K (07)** in MBs-representation]
Bernstein polynomials [**Liuti et. al (07)**]

dynamical models: not applied **[Radyuskin et.al (02); Tiburzi et.al (04); Hwang DM (07)]…**

flexible models: any representation by including ***unconstrained*** degrees of freedom
(for fits) **KMP-K (07/08)** for H1/ZEUS in MBs-representation

What is the physical content of ***`invisible'*** (***unconstrained***) degrees of freedom?

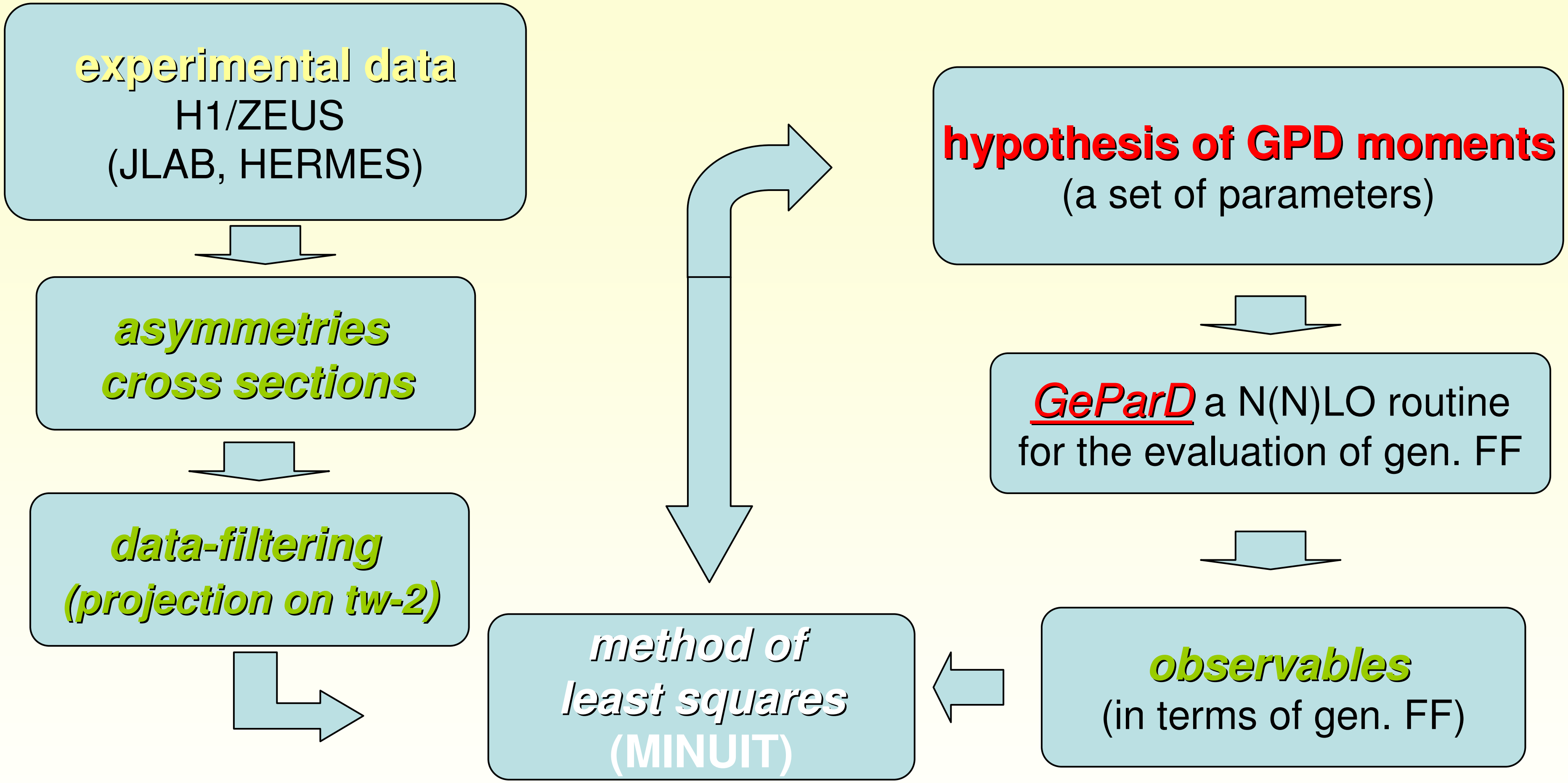
Extracting CFFs from data: real and imaginary part

0. analytic formulae [BMK 01]

i. (almost) without modeling **[Guidal, Moutarde (08/09)]**

ii. dispersion integral fits **[KMP-K (08),KM (08/09)]**
iii. flexible GPD modeling **[KM (08/09)]**

Ready for flexible GPD model fits?



the answer is ***YES*** for small x and ***NO*** for JLAB@6GeV kinematics:

- reasonable well motivated hypotheses of GPDs (moment) must be implemented
- many parameters – Is a least square fit an appropriate strategy?
- some technical, however, straightforward work is left (reevaluation of observables)

DVCS fits for H1 and ZEUS data

DVCS cross section measured at small $x_{\mathrm{Bj}} \approx 2\xi = \frac{2\mathcal{Q}^2}{2W^2+\mathcal{Q}^2}$

$$40\mathrm{GeV} \lesssim W \lesssim 150\mathrm{GeV}, \quad 2\mathrm{GeV}^2 \lesssim \mathcal{Q}^2 \lesssim 80\mathrm{GeV}^2, \quad |t| \lesssim 0.8\mathrm{GeV}^2$$

predicted by

$$\frac{d\sigma}{dt}(W,t,\mathcal{Q}^2) \approx \frac{4\pi\alpha^2}{\mathcal{Q}^4}\frac{W^2\xi^2}{W^2+\mathcal{Q}^2}\left[|\mathcal{H}|^2 - \frac{\Delta^2}{4M_\mathrm{p}^2}|\mathcal{E}|^2 + \left|\widetilde{\mathcal{H}}\right|^2\right](\xi,t,\mathcal{Q}^2)\Big|_{\xi=\frac{\mathcal{Q}^2}{2W^2+\mathcal{Q}^2}}$$

suppressed contributions *<<0.05>>* relative *O(ξ)*

- LO data are described with - huge (wrong) *t*-slope **[Belitsky, DM, Kirchner (01)]**
  - inconsistent GPDs **[Freund, McDermott, Strikman (03)]**
  - missing factor of ¼ **[Guzey, Teckentrup (06,08)]**

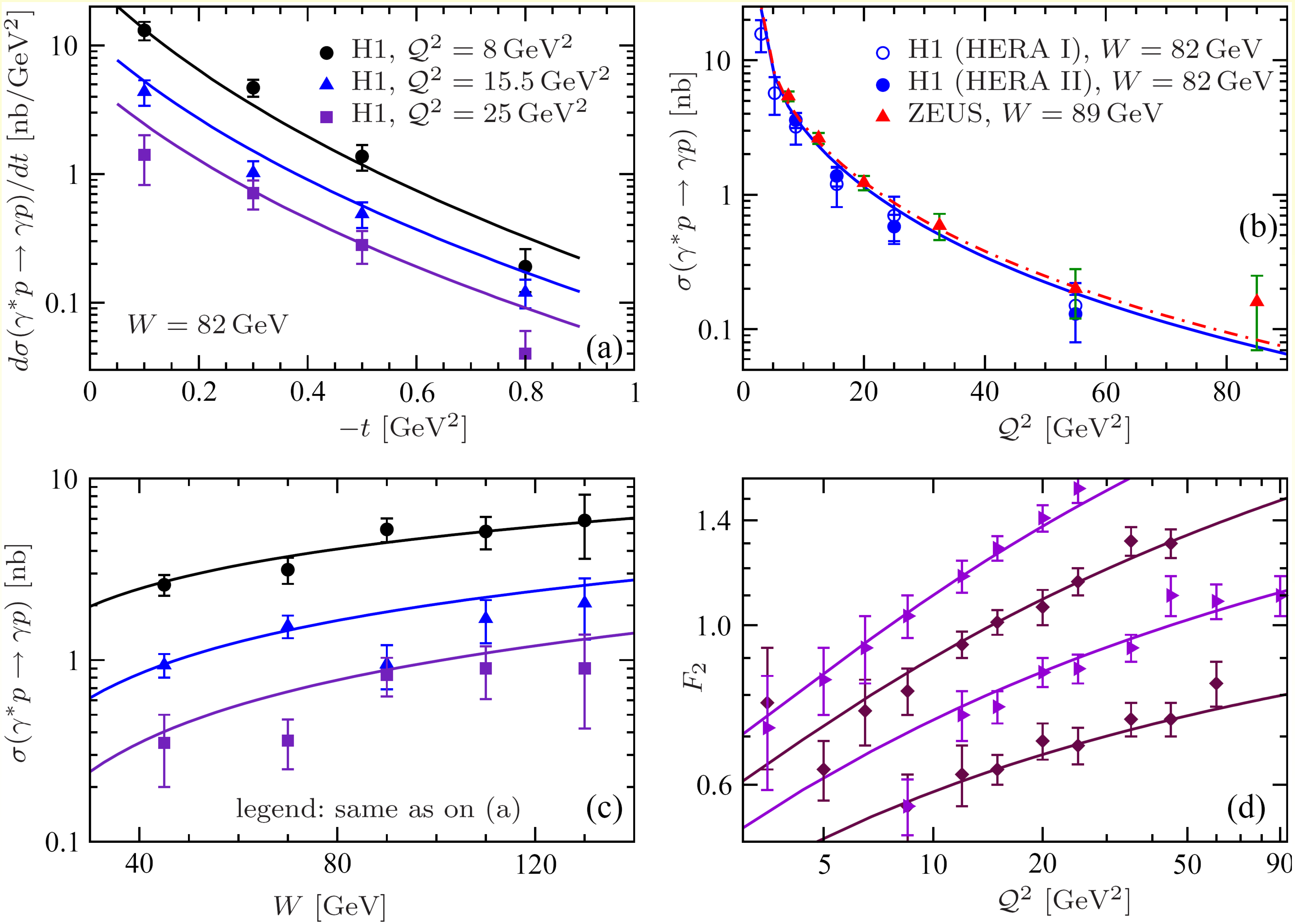
- NLO works with ad hoc GPD models **[Freund, McDermott (02)]**

  results strongly depend on employed PDF parameterization

⇨ ***do a simultaneous fit to DIS and DVCS*** **[KMP-K (07)]**

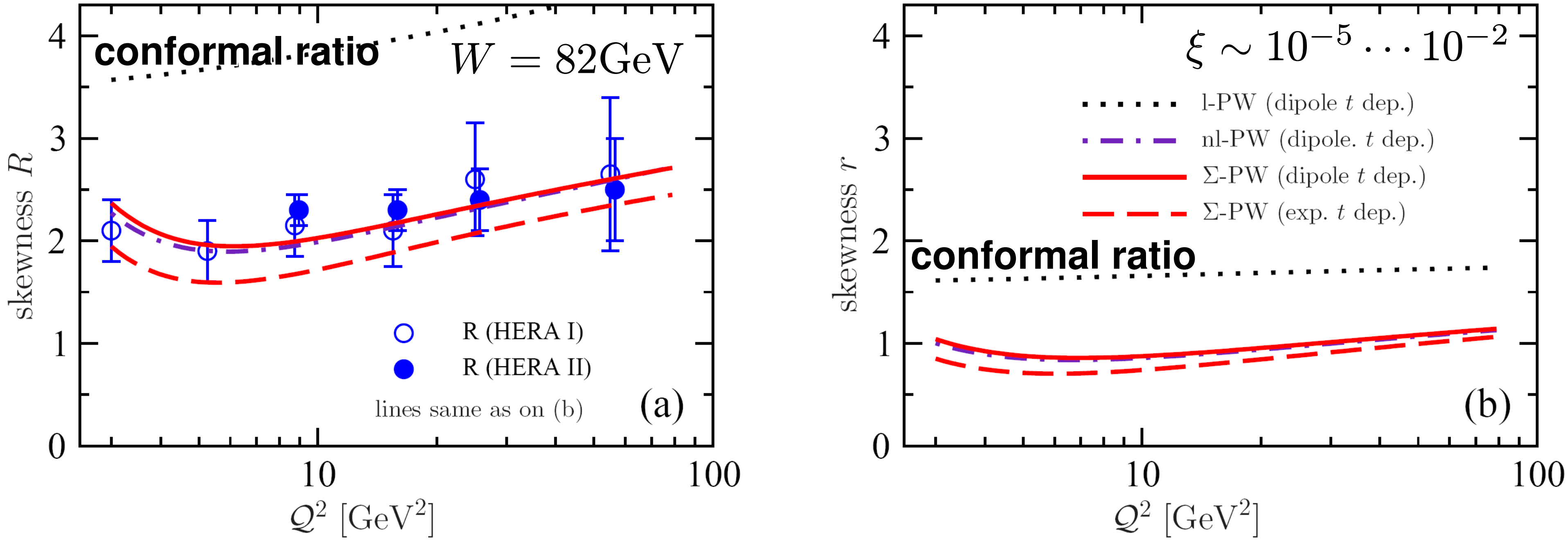
⇨ ***use flexible GPD models in a two-step fit*** **[KMP-K (08)]**

good DVCS fits at LO, NLO, and NNLO with flexible GPD ansatz



quark skewness ratio from DVCS fits @ LO

$$R = \frac{\Im\mathrm{m}A_{\mathrm{DVCS}}}{\Im\mathrm{m}A_{\mathrm{DIS}}} \overset{LO}{=} \frac{H(\xi,\xi)}{H(2\xi,0)} \approx 2^{\alpha} r \qquad r = \frac{H(\xi,\xi)}{H(\xi,0)}$$



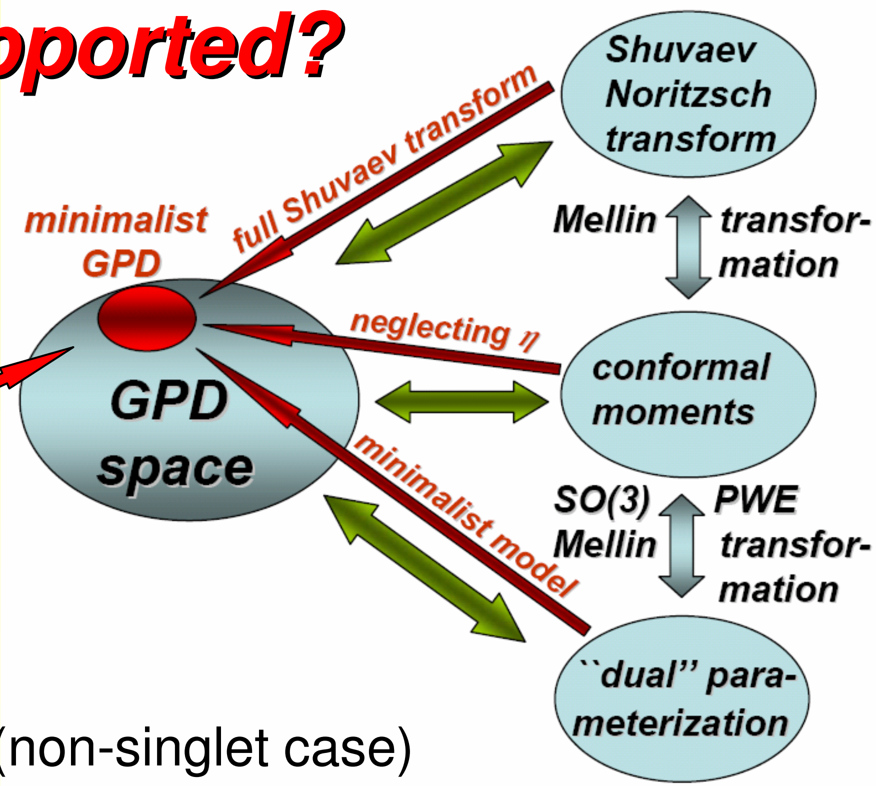
- @LO the conformal ratio is ruled out for sea quark GPD
- a generically zero-skewness effect over a large Q^2 lever arm
- scaling violation consistent with pQCD prediction
- this zero-skewness effect is non-trivial to realize in conformal space
  (SO(3) sibling poles are required)

Is the conformal ratio supported?

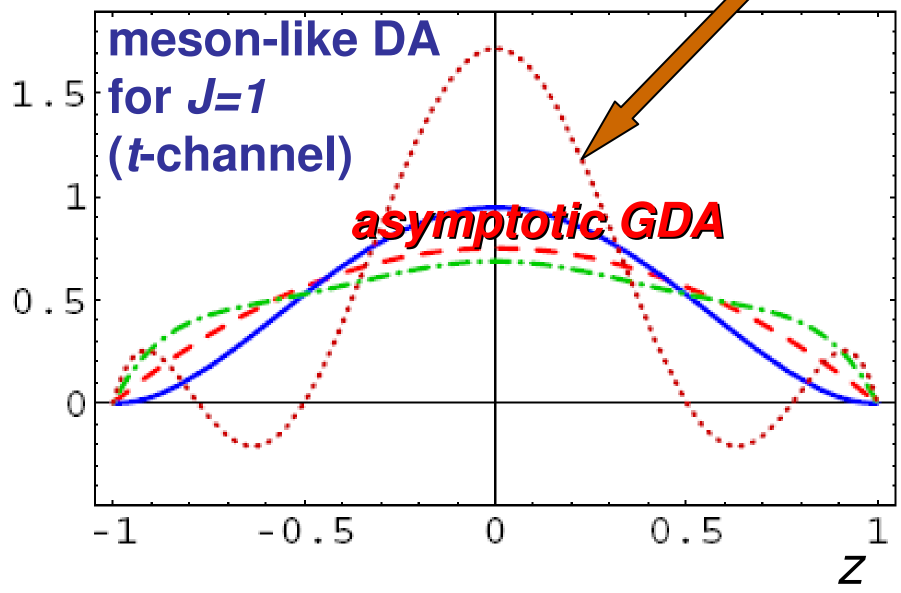
$$r = \frac{H(x,x,t=0,Q^2)}{q(x,Q^2)}$$

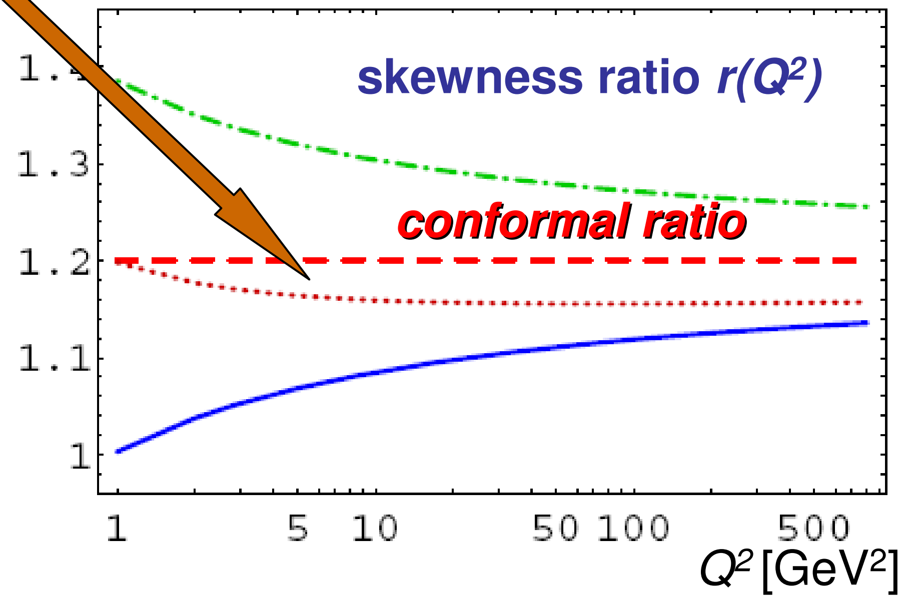
``erroneous small *x*-claim''

$$r_{\rm con} = \frac{2^\alpha \Gamma(3/2+\alpha)}{\Gamma(3/2)\Gamma(2+\alpha)}$$



a **counter example** (non-singlet case)

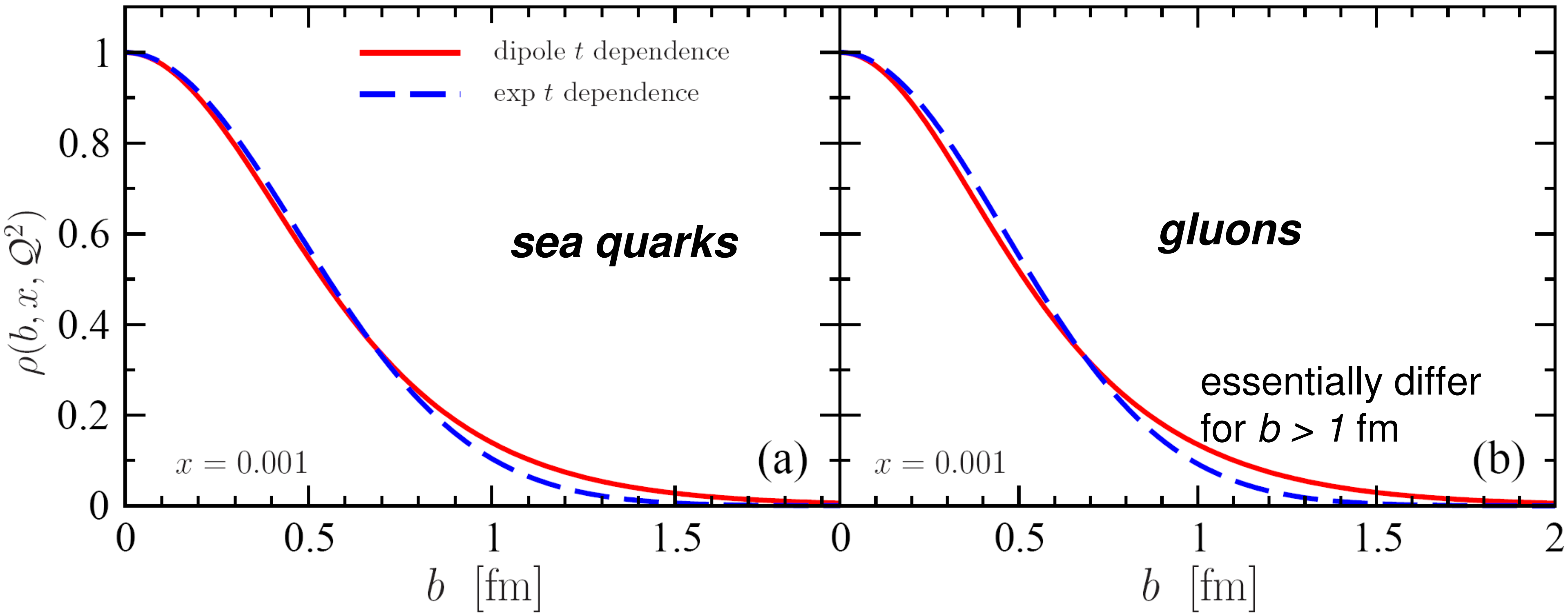




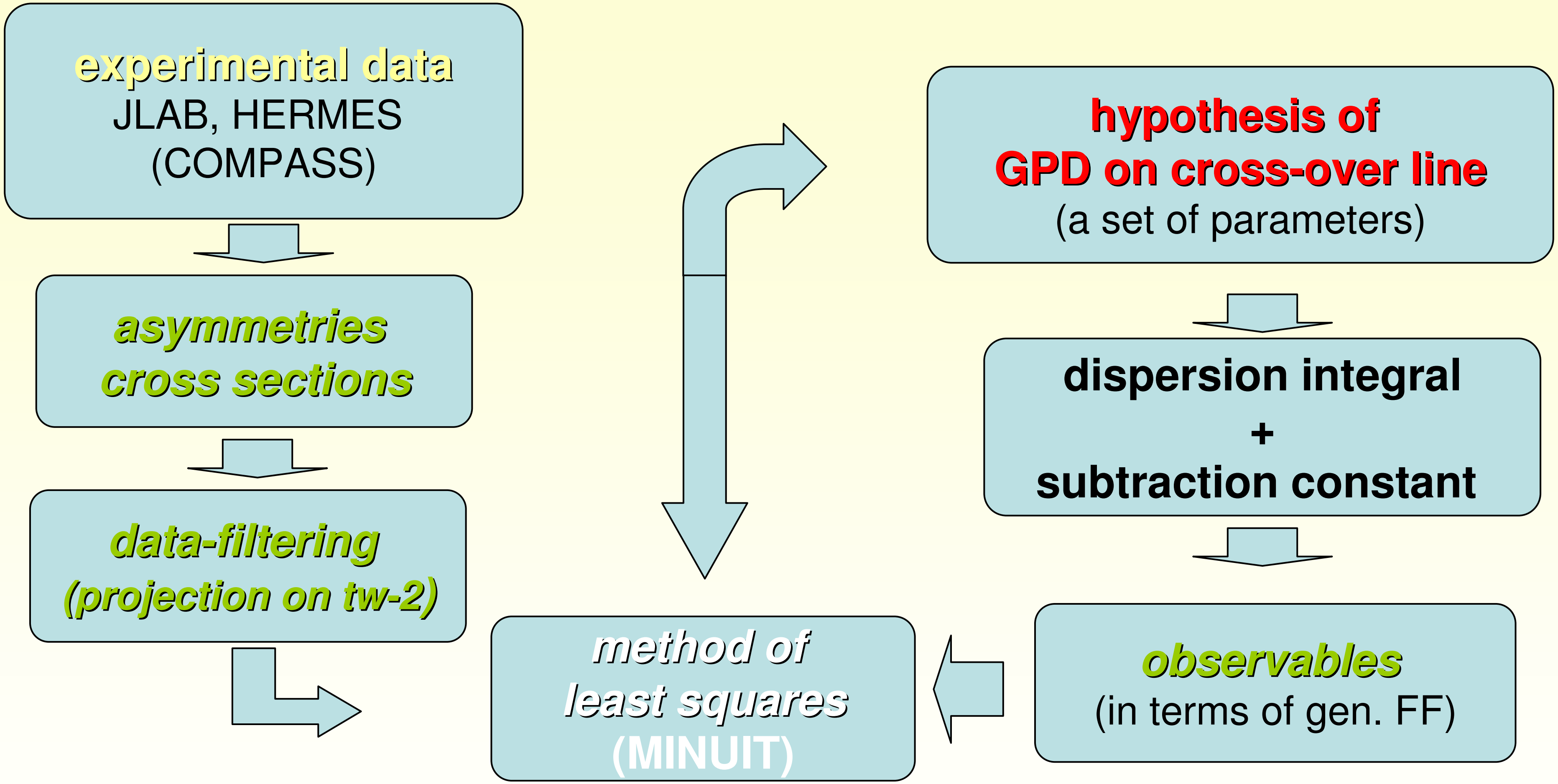
- CFF *H* posses ``pomeron behavior'' $\xi^{-\alpha(Q)\ -\ \alpha'(Q)t}$
  - ✓ ***α* increases with growing *Q^2***
  - ✓ ***α'* decreases growing *Q^2***
- *t*-dependence: exponential   shrinkage is disfavored   *($\alpha' \approx 0$)*

  dipole   shrinkage is visible *($\alpha' \approx 0.15$ at $Q^2=4$ GeV^2)*
- (normalized) profile functions

$$\rho \propto \int d^2\vec{\Delta}_\perp \, e^{i\vec{b}\cdot\vec{\Delta}_\perp} H(x,0,t=-\vec{\Delta}_\perp^2)$$



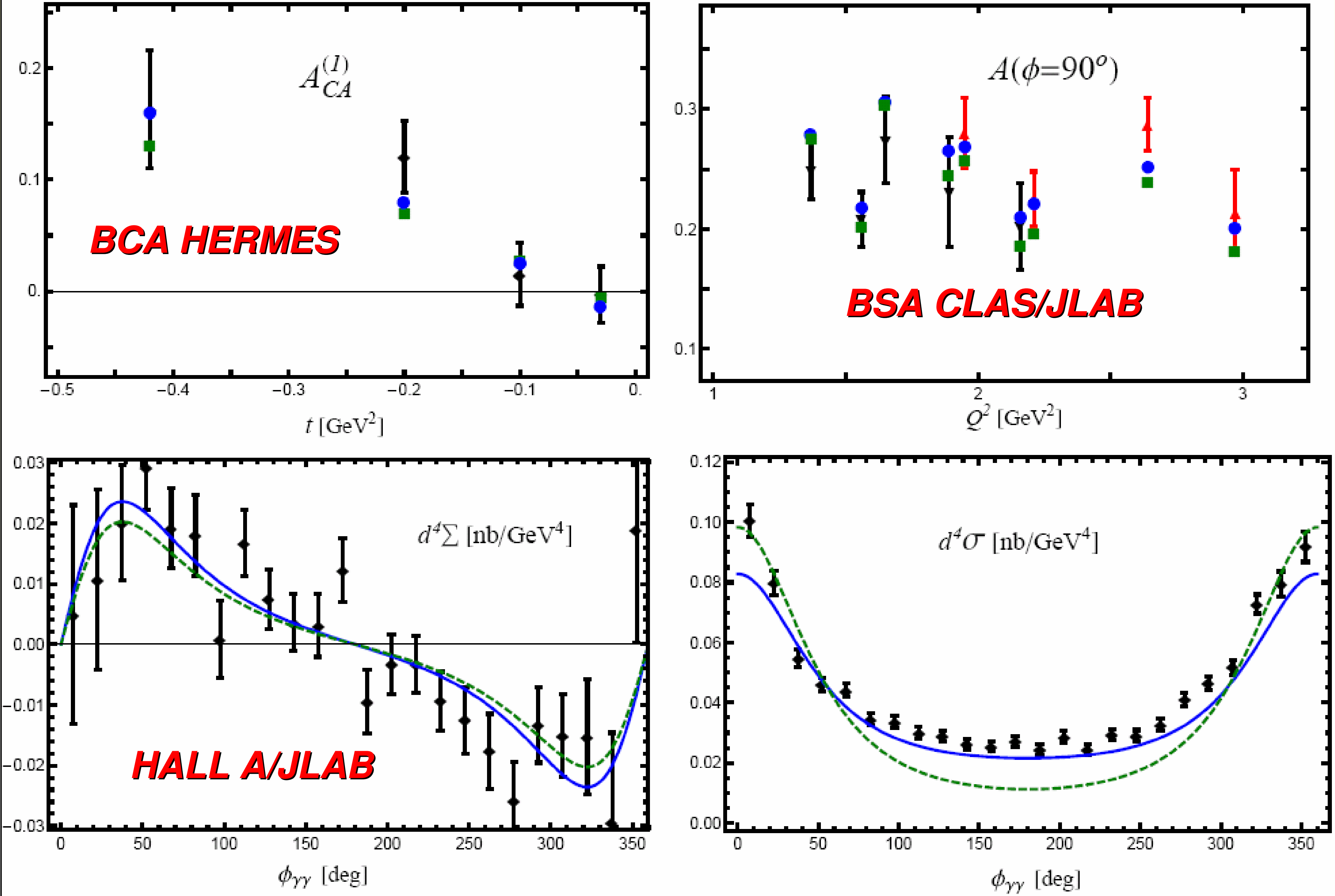
Ready for dispersion relation fits



the answer is *YES*, ***however, more data are needed:***

- to pin down the GPD models (on the cross over line $\eta = x$)
- to overcome the hypotheses of H (and twist-two) dominance
- relying on ***scaling hypothesis***

Global GPD fit example: HERMES & JLAB



• model of GPD *H(x,x,t) within DD motivated ansatz at Q^2=2* GeV^2

fixed: *PDF normalization* *eff. Reage pole* *large t-counting rules*

$$H(x,x,t) = \frac{n\, r\, 2^{\alpha}}{1+x} \left(\frac{2x}{1+x}\right)^{-\alpha(t)} \left(\frac{1-x}{1+x}\right)^{b} \frac{1}{\left(1 - \frac{1-x}{1+x}\frac{t}{M^2}\right)^{p}}.$$

free: *r-ratio at small x* *large x-behavior* *p-pole mass*

sea quarks (taken from LO fits)

$$n = 0.68,\ \ r = 1,\ \ \alpha(t) = 1.13 + 0.15t/\mathrm{GeV}^2,\ \ m^2 = 0.5\mathrm{GeV}^2,\ \ p = 2$$

valence quarks

$$n = 1.0,\ \ \alpha(t) = 0.43 + 0.85t/\mathrm{GeV}^2,\ \ p = 1$$

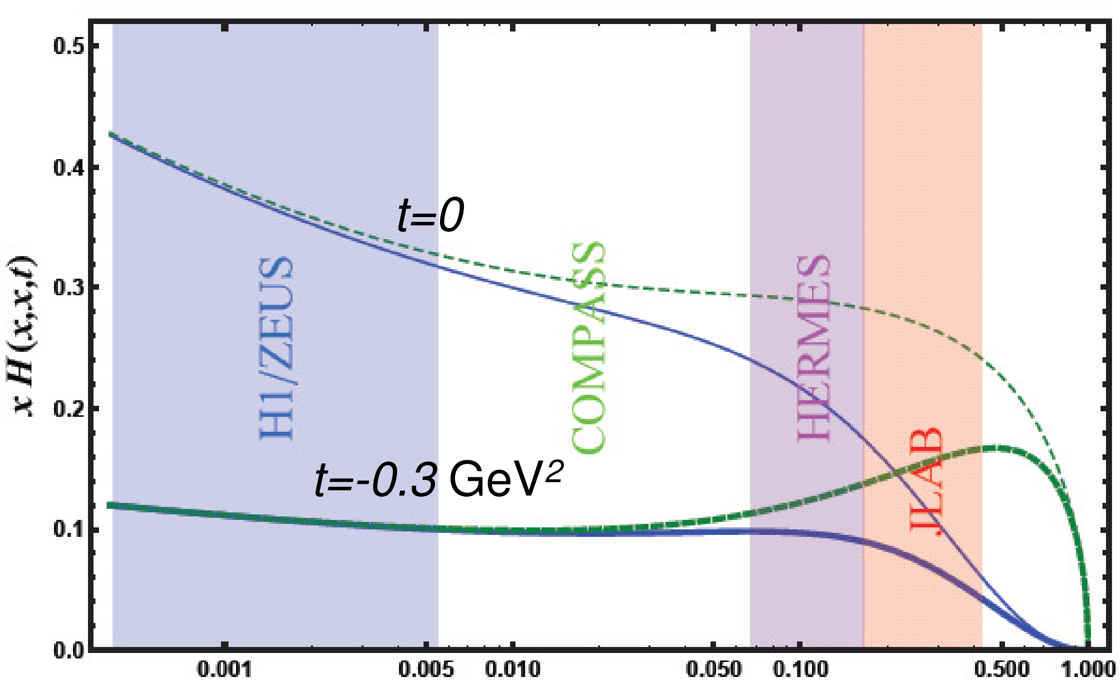
flexible parameterization of subtraction constant $\mathcal{D}(t) = \frac{-C}{(1-t/M_c^2)^2}$

+ pion-pole contribution

36 + 4 data points quality of ***global fit*** is good $\chi^2/\mathrm{d.o.f.} \approx 1$

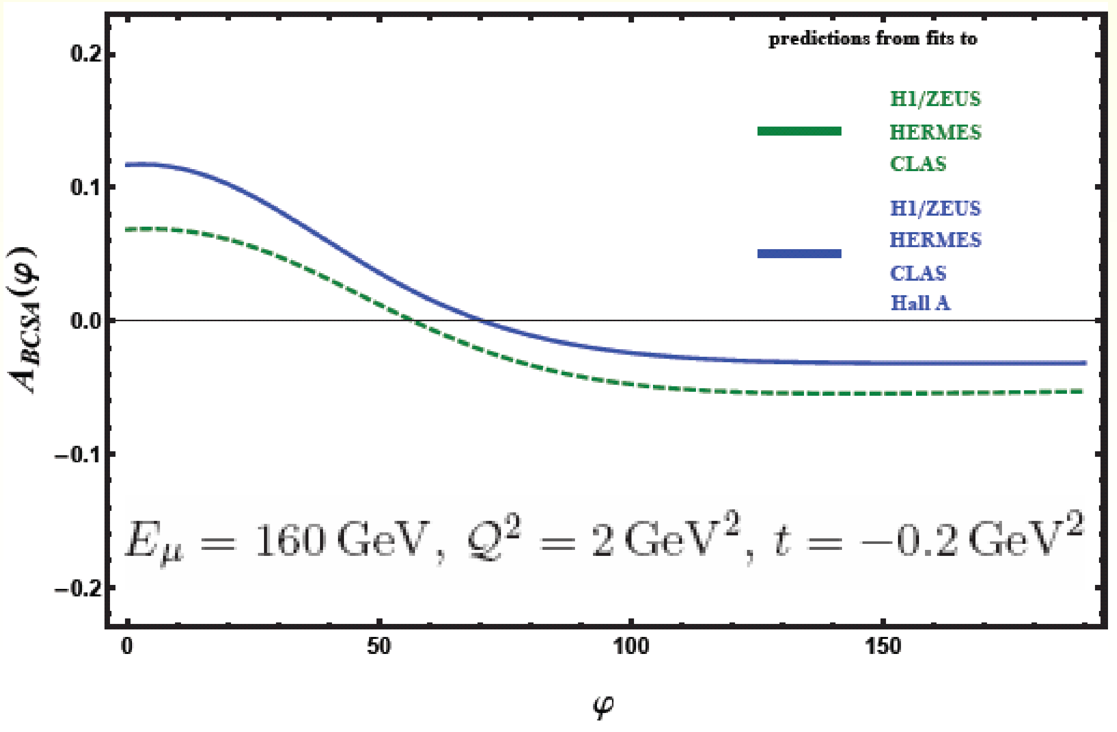
- extracting GPD

H(x,x,t,Q²=2 GeV²)



- prediction for COMPASS

$$A_{\mathrm{BCSA}} = \frac{d\sigma^{\uparrow +} - d\sigma^{\downarrow -}}{d\sigma^{\uparrow +} + d\sigma^{\downarrow -}}$$



Summary

GPDs are intricate and (thus) a promising tool

- to reveal the transverse distribution of partons
- to address the spin content of the nucleon
- providing a bridge to non-perturbative methods (e.g., lattice)

hard photon leptoproduction (DVCS)

- possesses a rich structure, allowing to access various CFFs/GPDs
- it is elaborated at twist-three (partly NLO) and NNLO
- it is widely considered as a theoretical clean process (supported by our scaling findings)

compatible strategies to analyze DVCS data

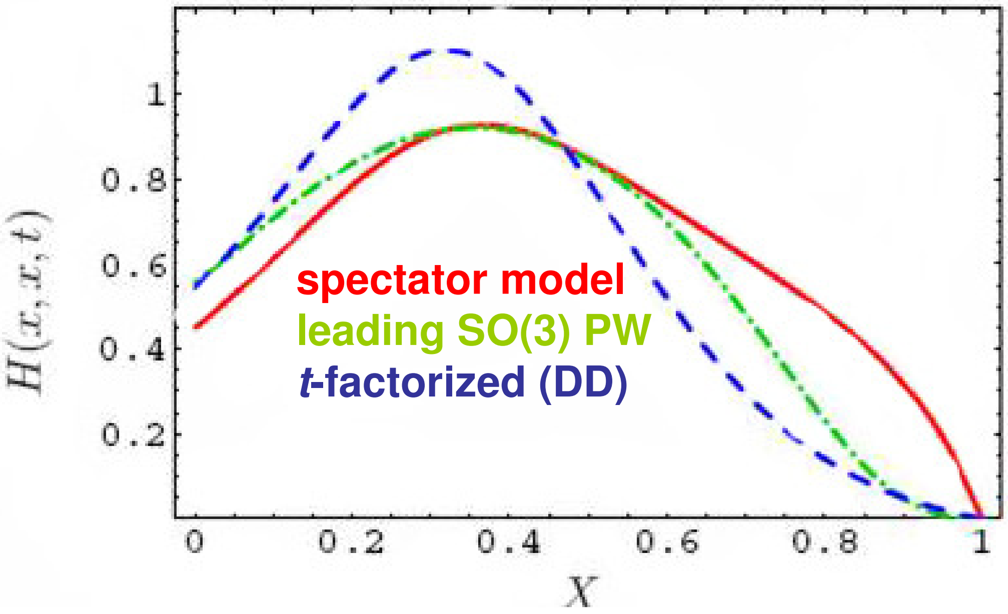
- analytic formulae, fitter code to extract CFFs
- flexible GPD models + fitting (minimizing X^2)
- dispersion integral technique for fixed target experiments

code for global fits

Back up slides are coming

(partonic) `quantum' numbers in GPD representations

name	's-channel' variable	't-channel' variable
GPD	PMF x	PMF ratio η
DD	PMF y	PMF z
CPWE	conformal spin $j+2$	PMF ratio η
'forward-like' CPWE	forward-like PMF z	PMF ratio η
Mellin-Barnes CPWE	conformal spin $j+2$	PMF ratio η
'dual' CPWE	forward-like PMF z	$\rho = j+2-J$
'dual' Mellin-Barnes CPWE	conformal spin $j+2$	t-channel AM J
SO(3)-PWE	PMF x	t-channel AM J



? about representation is not so essential

should be replaced by

How a GPD looks like on its cross-over trajectory ?

SL(2,R) representations for GPDs

- support is a consequence of Poincaré invariance (polynomiality)

$$H_j(\eta,t,\mu^2) = \int_{-1}^{1} dx\, c_j(x,\eta) H(x,\eta,t,\mu^2)\,, \qquad c_j(x,\eta) = \eta^j C_j^{3/2}(x/\eta)$$

- conformal moments evolve autonomous (to LO and beyond in a special scheme)

$$\mu \frac{d}{d\mu} H_j(\eta,t,\mu^2) = -\frac{\alpha_s(\mu)}{2\pi}\gamma_j^{(0)} H_j(\eta,t,\mu^2)$$

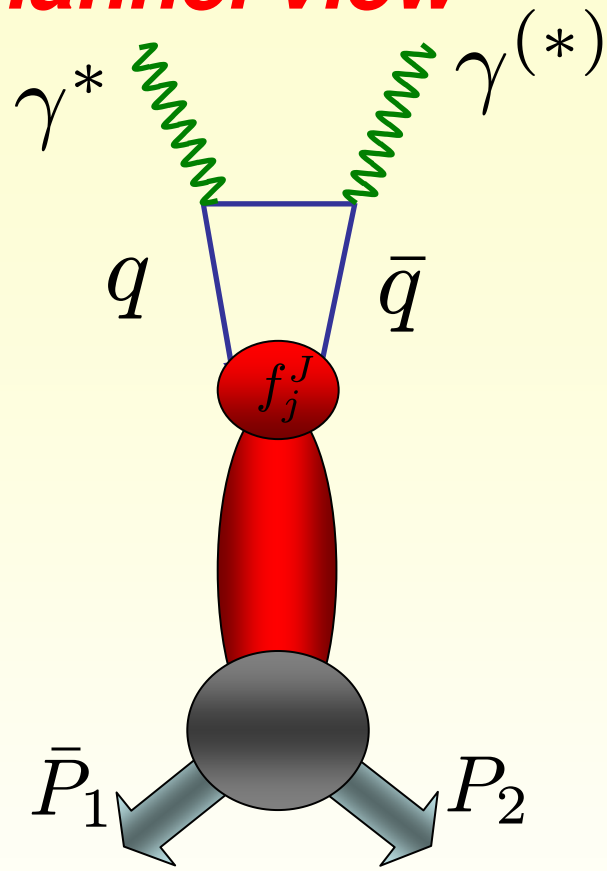
- inverse relation is given as series of mathematical distributions:

$$H(x,\eta,t) = \sum_{j=0}^{\infty}(-1)^j p_j(x,\eta) H_j(\eta,t)\,,\ p_j(x,\eta) \propto \theta(|x|\le\eta)\frac{\eta^2-x^2}{\eta^{j+3}} C_j^{3/2}(-x/\eta)$$

- various ways of resummation were proposed:

- smearing method [Radyushkin (97); Geyer, Belitsky, DM., Niedermeier, Schäfer (97/99)]
- mapping to a kind of forward PDFs [A. Shuvaev (99), J. Noritzsch (00)]
- dual parameterization (a mixture of both) [M. Polyakov, A. Shuvaev (02)]
- based on conformal light-ray operators [Balitsky, Braun (89); Kivel, Mankewicz (99)]
- ***Mellin-Barnes integral*** [DM, Schäfer (05); A. Manashov, M. Kirch, A. Schäfer (05)]

GPD ansatz at small x from t-channel view



- at short distance a quark/anti-quark state is produced, labeled by ***conformal spin*** *j+2*

- they form an intermediate mesonic state with total angular momentum ***J***
  strength of ***coupling*** is $f_j^J,\ J \leq j+1$

- mesons propagate with $\frac{1}{m^2(J)-t} \propto \frac{1}{J-\alpha(t)}$

- decaying into a nucleon anti-nucleon pair with given angular momentum *J,*
  described by an ***impact form factor***

$$F_j^J(t) = \frac{f_j^J}{J-\alpha(t)} \frac{1}{(1-\frac{t}{M^2(J)})^p}$$

! GPD *E* is zero if chiral symmetry holds
(partial waves are Gegenbauer polynomials with index 3/2)

D-term arises from the SO(3) partial wave *J=j+1* *(j* → *-1)*

Can the skewness function be constrained from lattice ?

- relation among measurable and GPD Mellin moments at *η=0*:

$$\int_0^1 d\xi\, \xi^j \Im\mathrm{m}\mathcal{F}(\xi, t, \mathcal{Q}^2) \stackrel{\mathrm{LO}}{=} \pi f_j(t, \mathcal{Q}^2)\left[1 + \delta_j(t, \mathcal{Q}^2)\right]$$

- deviation factors: $\delta_j(t, \mu^2) = \dfrac{\int_0^1 dx\, x^j S(x, t, \mu^2) F(x, \eta = 0, t, \mu^2)}{\int_0^1 dx\, x^j F(x, \eta = 0, t, \mu^2)}$

are given by a series of operator expectation values with increasing spin *j+n+1*

$$\delta_j(t, \mu^2) = \sum_{\substack{n=2\\ \mathrm{even}}}^{\infty} \frac{f^{(n)}_{j+n}(t, \mu^2)}{f_j(t, \mu^2)}\,, \qquad f^{(n)}_j(t, \mu^2) = \frac{1}{n!} \frac{d^n}{d\eta^n} f_j(\eta, t, \mu^2)\Big|_{\eta=0}$$

- lattice can evaluate *j=0,1,2,(3)*, i.e., *n=2:* $\delta_0(t, \mu^2 = 4\,\mathrm{GeV}^2) \approx 0.2 + ?$ thanks to **Ph. Hägler**

- ? wrong expectation from evolution: $\delta_j \sim \dfrac{2^{j+1}\Gamma(5/2 + j)}{\Gamma(3/2)\Gamma(3 + j)} - 1$

the analog small *x* prediction is ruled out **[Shuvaev et al. (99)]**

$$\delta_0 \sim 0.5 \qquad \delta_1 \sim 1.5$$